Partial supersymmetry breaking in N = 4 supergravity

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PARTIAL SUPERSYMMETRY BREAKING IN $N = 4$ SUPERGRAVITY

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It is shown that gauged $N = 4$ supergravity theories with matter can exhibit breaking to $N = 1$, 2 or 3 supersymmetries. Examples are constructed, all of which turn out to have a nonzero cosmological constant.

Partial supersymmetry breaking in extended supergravity theories ($N \geqslant 2$) is of potential importance for phenomenological applications of supergravity (see ref. [1] and references therein). In this paper, we consider this partial super-Higgs effect in $N = 4$ supergravity. In pure gauged $N = 4$ supergravity [2–4] only completely broken or unbroken supersymmetry is possible. Recently, the complete matter coupling in $N = 4$ was constructed by superconformal methods [5–7], and in ref. [7] we showed that with a single vector multiplet partial breaking cannot be achieved. Here we construct examples with more matter multiplets in which unbroken $N = 1$, 2 or 3 supersymmetry is realized. In all cases the cosmological constant is negative. The same holds for the known examples in $N = 3$ supergravity [8,9]. It has been shown [10,11] that in $N = 2$ supergravity one must go outside the framework of superconformal tensor calculus to obtain partial breaking in Minkowski space.

Let us first recall some properties of the fields of $N = 4$ supergravity with matter. There are two sets of scalar fields. The (pseudo) scalar fields of the supergravity multiplet parametrize the manifold SU(1,1)/U(1). They occur as an SU(1,1) doublet ϕ_α ($\alpha = 1,2$) and satisfy the SU(1,1) invariant condition ($\phi^1 \equiv (\phi_1)^*$, $\phi^2 \equiv -(\phi_2)^*$)

$$\phi^\alpha \phi_\alpha = 1 . \tag{1}$$

The scalars from the matter multiplets parametrize $SO(6,n)/SO(6) \times SO(n)$. They transform under the (real) six-dimensional representation of SU(4), are denoted by $\phi_{ij}{}^I$ ($i,j = 1, ..., 4; I = 1, ..., n+6$), with $\phi_{ij}{}^I = -\phi_{ji}{}^I$, $\phi^{ijI} \equiv (\phi_{ij}{}^I)^* = -\frac{1}{2}\epsilon^{ijkl}\phi_{kl}{}^I$ and satisfy the SO(6,n) invariant condition

$$\phi_{ij}{}^I \eta_{IJ} \phi^{klJ} = -\tfrac{1}{2}\delta^k_{[i}\delta^l_{j]} , \tag{2}$$

where η is the diagonal metric

$$\eta_{IJ} = \mathrm{diag}(-1,-1,-1,-1,-1,-1,+1, ..., +1) . \tag{3}$$

The matter multiplets also contain vector fields $A_\mu{}^I$ (six of which provide the vector fields of the supergravity multiplet), and spin-$\frac{1}{2}$ fields $\psi_i{}^I$, which transform as a **4** of SU(4). The spin-$\frac{1}{2}$ fields satisfy

$$\phi_{ij}{}^I \eta_{IJ} \psi_k{}^J = 0 . \tag{4}$$

The construction of this theory, which clarifies the origin of the conditions (2) and (4), can be found in ref. [5]. Due to (2) and (4) there are $6n$ physical scalars and $4n$ independent spin-$\frac{1}{2}$ fields associated with the matter multiplets, as well as n vector fields ($I = 7, ..., n+6$). The other physical fields are the remaining six vectors ($I = 1, ..., 6$), vierbein and gravitini, and four additional spin-$\frac{1}{2}$ fields Λ_i.

Subgroups G of the global symmetry group SO(6,n) may be gauged, and all matter fields labelled by I then transform according to the adjoint representation of G. The metric η_{IJ} must be invariant under G.

There exist inequivalent versions of pure gauged $N = 4$ supergravity. Two are based on the SO(4) symmetric theory, and have stationary points with unbroken [2] or completely broken [3] supersymmetry. The difference between these two can be traced back to a relative sign between the coupling constants of the two SO(3) factors in the SO(4) gauge group. A

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third version [4] has the SU(4) symmetric $N = 4$ theory as a starting point, and its potential does not have a stationary point. Inequivalent theories also appear in gauged $N = 4$ supergravity with matter. In our construction these differences arise in the following way. Each matter multiplet, labelled by I, is coupled to the $N = 4$ Weyl multiplet [12], which contains the SU(1,1) doublet ϕ. For each value of I one may introduce a parameter α_I [7], and replace ϕ by [‡1]

$$\begin{pmatrix}\phi_{1(I)}\\ \phi_{2(I)}\end{pmatrix} \equiv \begin{pmatrix}\exp(-\mathrm{i}\alpha_I) & 0\\ 0 & \exp(\mathrm{i}\alpha_I)\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}. \tag{5}$$

For ungauged theories different choices of α_I lead to classically equivalent versions. Gauging a semi-simple group G requires modifications, nonlinear in the scalar fields, both in the action and transformation rules, and this causes the inequivalence of models constructed with different values of α_I. The gauging also restricts the possible values of α_I: they must be the same for those values of I which correspond to the same simple factor of G. The possibility of choosing different α_I and coupling constants for the simple factors of G, allows for the construction of inequivalent gauged versions of equivalent ungauged theories.

The super-Higgs effect occurs in a stationary point of the theory if the matter fields at this point are not invariant under a supersymmetry transformation. In a constant background without fermion condensates this means that

$$\langle\delta\chi_i\rangle = \langle A_{ij}\rangle\,\epsilon^j \neq 0\,, \tag{6}$$

where χ_i is a generic spin-$\frac{1}{2}$ field, and A_{ij} a function of the scalar fields. Partial breaking occurs whenever $\langle A_{ij}\rangle$ has one or more zero eigenvalues.

There are a number of observations which facilitate the search for partial breaking. The first is, that it is sufficient to look for constant scalar field configurations for which A_{ij} has zero eigenvalue(s). Such field configurations automatically correspond to stationary points of the scalar potential, V [10]. This means that in the search for stationary points with unbroken supersymmetry we need not consider V itself, but only the variation of the spin-$\frac{1}{2}$ fields. Since we are mainly interested in providing examples of partial breaking, we will simplify the analysis even further by considering only configurations which have a non-trivial invariance group. These techniques have also been employed to establish the existence of $N = 1$ supersymmetric stationary points in matter coupled $N = 3$ supergravity theories [9].

With these observations we embark on the analysis of the super-Higgs effect in $N = 4$ theories. The transformation rules of the fermions take the form (non-derivative contributions of scalar fields only):

$$\delta\Lambda_i = -\tfrac{4}{3}\eta_{IJ}\Phi^*_{(I)}X_{ij}{}^{IJ}\epsilon^j\,, \tag{7}$$

$$\delta\psi_i{}^I = -2P^I{}_J\Phi^*_{(J)}W_i{}^{jJ}\epsilon_j\,, \tag{8}$$

$$\delta\psi_\mu{}^i = -\tfrac{2}{3}\gamma_\mu\eta_{IJ}\Phi^*_{(I)}X^{ijIJ}\epsilon_j\,, \tag{9}$$

where we have defined

$$W_i{}^{jI} = f_{KL}{}^I\phi_{ik}{}^K\phi^{kjL} \quad (\mathbf{15}\text{ of SU(4)})\,, \tag{10}$$

$$X_{ij}{}^{IJ} = f_{KL}{}^J\phi^{klI}\phi_{ik}{}^K\phi_{lj}{}^L \quad (\mathbf{10}\text{ of SU(4)})\,, \tag{11}$$

$$P^I{}_J = \delta_J{}^I + \eta_{JK}\phi_{ij}{}^I\phi^{ijK} \quad (P^2 = P, P\phi_{ij} = 0)\,, \tag{12}$$

$$\Phi_{(I)} = \exp(\mathrm{i}\alpha_I)\,\phi^1 + \exp(-\mathrm{i}\alpha_I)\,\phi^2\,. \tag{13}$$

The structure constants of G, $f_{KL}{}^I$, are real, and satisfy

$$f_{KL}{}^I\eta_{IJ} + f_{KJ}{}^I\eta_{LI} = 0\,, \tag{14}$$

because of the invariance of η under G. The scalar potential reads

$$V = \tfrac{4}{9}|\eta_{IJ}\Phi^*_{(I)}X_{ij}{}^{IJ}|^2 - \tfrac{1}{2}\eta_{IJ}|\Phi_{(I)}|^2 W_i{}^{jI}W_j{}^{iJ}\,. \tag{15}$$

There are mass terms for the fermions, of which we need only those in which the gravitino plays a role:

$$e^{-1}\mathcal{L}_{\mathrm{m,grav.}} = -\tfrac{2}{3}\eta_{IJ}\Phi^*_{(I)}X^{ijIJ}\bar\psi_{\mu i}\sigma^{\mu\nu}\psi_{\nu j} + \bar\psi_{\mu i}\gamma^\mu\eta^i + \text{h.c.}\,, \tag{16}$$

where the goldstino η^i is given by

$$\eta^i = \tfrac{1}{3}\eta_{IJ}\Phi_{(I)}X^{ijIJ}\Lambda_j + \eta_{IJ}\Phi^*_{(I)}W_j{}^{iI}\psi^{jJ}\,. \tag{17}$$

It is instructive to consider the supersymmetry transformation of η^i. In a constant bosonic background it reads

$$\delta\eta^i = -\tfrac{4}{9}(\eta_{IJ}\Phi_{(I)}X^{ijIJ})(\eta_{KL}\Phi^*_{(K)}X_{jk}{}^{KL})\,\epsilon^k + 2\eta_{IJ}\Phi^*_{(I)}W_j{}^{iI}P^J{}_K\Phi_{(K)}W_k{}^{jK}\epsilon^k\,. \tag{18}$$

[‡1] Actually, the 2 × 2 matrix can be extended to a full SU(1,1) matrix, but only the diagonal U(1) subgroup is relevant for this discussion.

Both contributions to (18) are of the form $A^{\dagger}A\epsilon$, so that $\delta\eta^i = 0$ implies both $\delta\Lambda_i = 0$ and $\delta\psi^{iI} = 0$. On the other hand, (18) can be rewritten in the form

$$\delta\eta^i = -[V\delta^i{}_j + \tfrac{4}{3}(\eta_{IJ}\Phi^*_{(I)}X^{ikIJ}) \times (\eta_{KL}\Phi_{(K)}X_{kj}{}^{KL})]\,\epsilon^j\,. \tag{19}$$

Therefore $\delta\eta^i = 0$ also implies that the gravitino mass matrix has an eigenvalue $m_{3/2} = (-V/3)^{1/2}$.

Let us assume for the moment that all angles α_I in (5) vanish, so that $\phi_{(I)}$ is independent of I (all α_I equal is equivalent to $\alpha_I \equiv 0$). Then we see that the condition $\langle\delta\Lambda_i\rangle = 0$ for an unbroken supersymmetry implies that

$$X_{ij} \equiv \eta_{IJ}X_{ij}{}^{IJ} \tag{20}$$

has an eigenvalue zero, and therefore that the gravitino mass matrix has eigenvalue zero for the same eigenvector. We then conclude from (19) that necessarily $V = 0$, i.e. unbroken supersymmetry with $\alpha_I \equiv 0$ can only occur in a Minkowski background. This can also be inferred from the form of the potential (15). For $\alpha_I \equiv 0$ the dependence on the scalar fields ϕ_α is contained in the factor $|\Phi|^2$. This factor is just the Freedman–Schwarz potential [4], which is known to have no stationary points in ϕ_α. So indeed a stationary point of V can only be achieved for configurations with

$$\tfrac{4}{9}X_{ij}X^{ij} = \tfrac{1}{2}\eta_{IJ}W_i{}^{jI}W_j{}^{iJ}\,, \tag{21}$$

or $V = 0$. The case with $\alpha_I \equiv 0$ corresponds to $N = 4$ supergravity theories with global SU(4) symmetry. We have not found any configurations with partial supersymmetry breaking in theories of this type.

Therefore we return to the general case, and analyze the variations (7) and (8). To exhibit more clearly the dependence on the scalar fields, and the effect of the condition (2), it is convenient to replace the complex variables $\phi_{ij}{}^I$ by real fields $x_p{}^I$ ($p = 1, ..., 6$), which transform according to the vector representation of SO(6). This is achieved by setting ($m = 1,2,3$)

$$\phi_{ij}{}^I = \tfrac{1}{2}(\beta_{ij}{}^m x_m{}^I + \mathrm{i}\alpha_{ij}{}^m x_{m+3}{}^I)\,, \tag{22}$$

where β_m and α_m are anti-symmetric, real, (anti-)selfdual 4×4 matrices, whose properties are given in e.g. ref. [13]. Then (2) and (4) take on the form

$$x_p{}^I\eta_{IJ}x_q{}^J = -\delta_{pq}\,, \quad x_p{}^I\eta_{IJ}\psi_i{}^J = 0\,. \tag{23}$$

In the search for partial supersymmetry breaking we restrict ourselves to configurations with an invariance group H. In fact, H must be a diagonal subgroup of SO(6) $\otimes$ G under which $x_p{}^I$ is invariant. H can be at most the maximal SO(4) subgroup of SO(6). Then the eigenvalues of the gravitino mass matrix are all equal, so that there are four broken or four unbroken supersymmetries. We therefore consider first H = SO(3), which is embedded in SU(4) in such a way that $\mathbf{4} \to \mathbf{3} \oplus \mathbf{1}$ for this subgroup. This implies that three eigenvalues of the gravitino mass matrix will be equal, so that only partial breaking to $N = 1$ and $N = 3$ is possible. The required configurations $x_p{}^I$ take on the form ($I = 3(a-1) + l$; $a = 1, ..., A$; $l = 1,2,3$)

$$x_m{}^I + \mathrm{i}x_{m+3}{}^I \equiv (u_a + \mathrm{i}v_a)\,\delta_{lm} \equiv z_a\delta_{lm}\,. \tag{24}$$

In terms of the variables z_a the constraint (23) corresponds to ($\eta_a = (-1,-1,+1, ..., +1)$):

$$\sum_{a=1}^{A} \eta_a(z_a)^2 = \sum_{a=1}^{A} \eta_a|z_a|^2 + 2 = 0\,, \tag{25}$$

which, for any z_a, $a \geqslant 3$, can be solved for z_1 and z_2. The gauge group we choose with these configurations is G = $[\mathrm{SO}(3)]^A$, with structure constants $f_{IJ}{}^K = \delta_{ab}\delta_{ac}g_a\epsilon_{lmn}$, where $I = 3(a-1) + l$, $J = 3(b-1) + m$, $K = 3(c-1) + n$, and g_a are arbitrary constants. This corresponds to the coupling of gauged $N = 4$ supergravity to an $N = 4$ Yang–Mills multiplet with gauge group $[\mathrm{SO}(3)]^{A-2}$. The calculation of $W_i{}^{jI}$ and $X_{ij}{}^{II}$ is a simple matter with this choice of G. Using a suitable basis for the matrices α_m and β_m of (22) we obtain

$$W_i{}^{jI} = \tfrac{1}{2}g_a\,[u_a^2\beta_l + v_a^2\alpha_l + \mathrm{i}u_av_a\epsilon_{lmn}\alpha_m\beta_n]_{ij}\,, \tag{26}$$

$$X_{ij}{}^{II} = \tfrac{3}{4}g_a\,\mathrm{diag}[|z_a|^2z_a, |z_a|^2z_a, |z_a|^2z_a, (z_a^*)^3]_{ij}. \tag{27}$$

The partial super-Higgs effect requires the existence of spinor(s) ϵ_j satisfying (see (7), (8)):

$$\eta_{IJ}\Phi^*_{(I)}X_{ij}{}^{IJ}\epsilon^j = 0 = P^I{}_J\Phi^*_{(J)}W_i{}^{jJ}\epsilon_j\,. \tag{28}$$

Using (12) and (24), we can write this last condition in the form

$$\Phi^*_aW_i{}^{j[3(a-1)+l]}\,\epsilon_j = u_a\xi_i{}^l + v_a\xi_i{}^{l+3}\,, \tag{29}$$

with arbitrary ξ. Recall that for values I which transform under the same simple factor of G, $\Phi^*_{(I)}$ is independent of I. Therefore, in the present case $\Phi^*_{[3(a-1)+l]}$ is independent of l and, by abuse of notation, will be called Φ_a^* from now on. Define the matrix

$$S = \begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \end{bmatrix} . \tag{30}$$

Its inverse exists because of the fact that z_1 and z_2 are constrained by (25). Solving ξ from (29) for $a = 1,2$ one obtains for $a \geqslant 3$

$$\left(\Phi_a^* W_i^{j[3(a-1)+l]} - [u_a v_a]\, S^{-1} \begin{bmatrix} \Phi_1^* W_i^{jl} \\ \Phi_2^* W_i^{j(l+3)} \end{bmatrix} \right) \epsilon_j = 0 . \tag{31}$$

After the substitution of (27), the first condition in (28) becomes

$$\sum_{a=1}^{A} g_a \Phi_a^* \eta_a \times \mathrm{diag}[|z_a|^2 z_a, |z_a|^2 z_a, |z_a|^2 z_a, (z_a^*)^3]_{ij}\, \epsilon^j = 0. \tag{32}$$

It is clear from (32) that partial breaking to $N = 1$ or $N = 3$ supersymmetry requires:

$$N = 1: \quad \sum_{a=1}^{A} g_a \Phi_a^* \eta_a (z_a^*)^3 = 0\,, \quad \epsilon_4 \text{ unbroken}\,, \tag{33a}$$

$$N = 3: \quad \sum_{a=1}^{A} g_a \Phi_a^* \eta_a |z_a|^2 z_a = 0\,, \quad \epsilon_1, \epsilon_2, \epsilon_3 \text{ unbroken}\,, \tag{33b}$$

The additional conditions that follow from (31) on substitution of (26) take the form ($a \geqslant 3$):

$$N = 1: \quad g_a \Phi_a^* (z_a)^2 = [u_a v_a]\, S^{-1} \begin{bmatrix} g_1 \Phi_1^* (z_1)^2 \\ g_2 \Phi_2^* (z_2)^2 \end{bmatrix}, \tag{34a}$$

$$N = 3: \quad g_a \Phi_a^* (z_a^*)^2 = [u_a v_a]\, S^{-1} \begin{bmatrix} g_1 \Phi_1^* (z_1^*)^2 \\ g_2 \Phi_2^* (z_2^*)^2 \end{bmatrix},$$

$$g_a \Phi_a^* |z_a|^2 = [u_a v_a]\, S^{-1} \begin{bmatrix} g_1 \Phi_1^* |z_1|^2 \\ g_2 \Phi_2^* |z_2|^2 \end{bmatrix}. \tag{34b}$$

At this point there are two ways to proceed. One may choose a set of coupling constants g_a and phase angles α_a, thus defining a theory, and solve (33), (34) for the scalar fields contained in Φ_a and z_a. On the other hand one may choose arbitrary, constant scalar field values in Φ_a and in z_a, satisfying the constraints (1) and (25), and show that coupling constants g_a and angles α_a exist for which (33) and (34) are satisfied. As we are presently proving existence of solutions, we follow the more convenient second route. A solution then defines a theory, of which the scalar field configuration ϕ_α, z_a corresponds to a stationary point. In solving (33) and (34) it is important to note that the combinations $g_a \Phi_a^*$ ($a = 1, ..., A$) which occur in W and X, can be given any complex value by choosing suitable g_a and α_a. Therefore we consider eqs. (33) and (34) as equations for the A unknown complex constants $g_a \Phi_a^*$, with given z_a satisfying (25).

Eqs. (34) are easily solved if one uses the simple fact that

$$z_a = [u_a v_a]\, S^{-1} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} . \tag{35}$$

For the $N = 1$ case this means that the general solution of (34a) is (for all $a = 1, ..., A$)

$$g_a \Phi_a^* (z_a)^2 = z_a C + z_a^* D\,, \tag{36}$$

where C and D are complex constants. The condition (33a) then imposes one linear relation between C and D:

$$C \sum_{a=1}^{A} \eta_a \frac{(z_a^*)^3}{z_a} + D \sum_{a=1}^{A} \eta_a \frac{(z_a^*)^4}{(z_a)^2} = 0\,. \tag{37}$$

Thus we have, given z_a, a one (complex) parameter solution of the $N = 1$ equations (33a), (34a) and therefore there exists a class of theories, with different g_a and α_a, which have the same scalar field configurations as a stationary point.

It is important to know the value of the scalar potential in the stationary point. To do this, one need only calculate the eigenvalue of the gravitino mass matrix associated with the unbroken supersymmetry. We find, using (16), (25), (27) and (36),

$$m_{3/2} = \frac{1}{2} \left| \sum_{a=1}^{A} g_a \Phi_a \eta_a (z_a^*)^3 \right| = |D|\,. \tag{38}$$

The value of the potential, which for the configurations (24) takes on the form

$$V=\frac{3}{4}\left|\sum_{a=1}^{A}\eta_a g_a \Phi_a^* |z_a|^2 z_a\right|^2+\frac{1}{4}\left|\sum_{a=1}^{A}\eta_a g_a \Phi_a^* (z_a^*)^3\right|^2$$
$$+\frac{3}{2}\sum_{a=1}^{A}\eta_a |g_a \Phi_a^* (z_a)^2|^2 , \tag{39}$$

indeed equals $-3|D|^2$ when (36) and (37) are satisfied. Clearly $V=0$ can only occur for $D=0$. However, in that case the remaining three eigenvalues of the gravitino mass matrix, which are proportional to $|D|$, vanish as well, and in this limiting case there are four unbroken supersymmetries. We will come back to the $N=4$ cases after we have treated the $N=2$ case.

In the $N=3$ case there are more equations than unknowns, but the equations turn out to be dependent. Using (35), one easily sees that eqs. (34b) are both satisfied if (for $a=1, ..., A$)

$$g_a \Phi_a^* z_a^* = M .$$

Then (33b) is also satisfied, because of the constraints (25). Also in this case the gravitino mass corresponding to the (three) unbroken supersymmetries does not vanish. From (27) and (16) we can read off that

$$m_{3/2}=\frac{1}{2}\left|\sum_{a=1}^{A} g_a \Phi_a \eta_a |z_a|^2 z_a\right| = |M| . \tag{40}$$

$M=0$ leaves all four supersymmetries unbroken.

With the same gauge group $G=[SO(3)]^A$ it is also possible to obtain partial breaking to $N=2$ supersymmetry. This requires a more general class of configurations than (24). If we choose

$I=3(a-1)+l, \quad l,m=1,2$:

$$x_m{}^I + \mathrm{i}x_{m+3}{}^I \equiv (u_a+\mathrm{i}v_a)\,\delta_{lm} \equiv z_a \delta_{lm} ,$$

$I=3a$:

$$x_3{}^I + \mathrm{i}x_6{}^I \equiv (u_{a3}+\mathrm{i}v_{a3}) \equiv z_{a3} , \tag{41}$$

then the SO(3) invariance group reduces to SO(2). The remainder of the analysis proceeds along similar lines, although the explicit forms of $W_i{}^{JI}$ and $X_{ij}{}^{IJ}$ differ from (26), (27). The result is that $N=2$ supersymmetry is preserved if the following conditions are satisfied for $a=1, ..., A$:

$$g_a \Phi_a^* z_{a3}^* = M \neq 0 , \quad |z_a| = |z_{a3}| . \tag{42}$$

The $N=2$ case also has a negative cosmological constant, since from (42) it follows that $m_{3/2}=|M|$. The class of solutions with $N=2$ supersymmetry contains of course the $N=3$ solutions obtained previously. The $N=2$ case reduces to $N=3$ whenever either

$$z_a = \pm z_{a3} \exp(\mathrm{i}\alpha) , \tag{43}$$

where α is an overall phase, but the sign may be chosen differently for each SO(3) group, or

$$z_a = \pm z_{a3}^* \exp(\mathrm{i}\beta) . \tag{44}$$

If both relations (43) and (44) hold, there are four unbroken supersymmetries. These two conditions also characterize the $N=4$ cases among the $N=1$ and $N=3$ configurations.

Note that $N=2$ requires $A \geqslant 4$. For $A=3$ the condition (42), together with the constraints (25), implies that (43) or (44) (or both) must hold, so that in fact a third supersymmetry is also unbroken.

In conclusion, we have shown that in $N=4$ supergravity stationary points with one, two or three unbroken supersymmetries can be realized in an anti-de Sitter background. It remains an open question, whether or not partial breaking is possible in Minkowski space.

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